2.3. Kirchhoff's Laws

- **Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node is zero. Or The sum of the currents entering a node is equal to the sum of the currents leaving the node.**

Mathematically, KCL implies that:

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node. For the circuit shown in Fig. 1.18

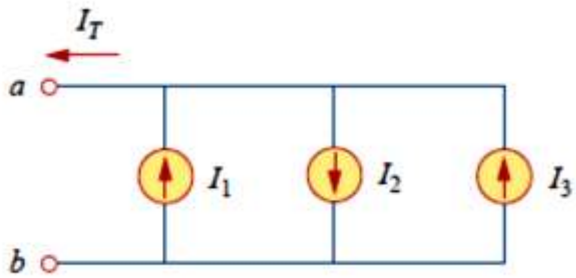


Figure 1.18: Current sources in parallel

$$I_T + I_2 = I_1 + I_3$$

Or:

$$I_T = I_1 - I_2 + I_3$$

- **Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.**

mathematically, KVL states that:

$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage. consider the circuit in Fig. 1.19.

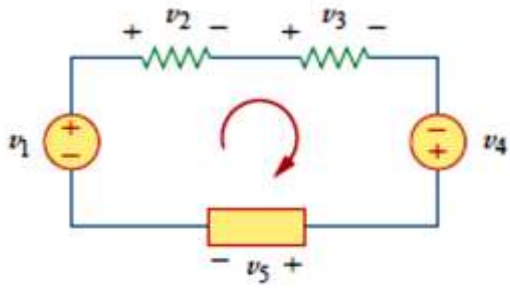


Figure 1.19: A single-loop circuit illustrating KVL.

The sign on each voltage is the **polarity of the terminal encountered first** as we travel around the loop. Thus, KVL yields:

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Rearranging terms gives:

$$v_2 + v_3 + v_5 = v_1 + v_4$$

which may be interpreted as:

Sum of voltage drops = Sum of voltage rises

Example 9: For the circuit in Fig. 1.20(a), find voltages v_1 and v_2.

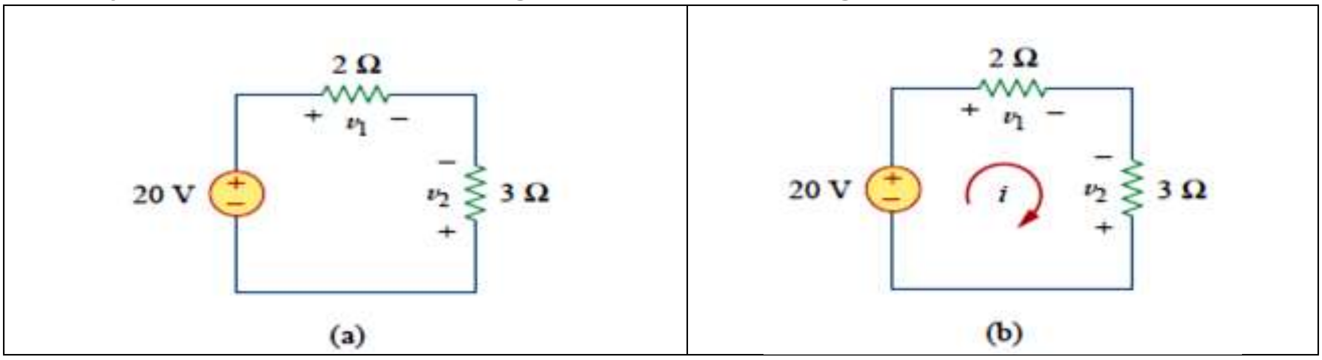


Figure 1.20: Circuit of Example 9.

<u>Solution:</u> To find v_1 and v_2 we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 1.20 (b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i$$

Applying KVL around the loop gives:

$$-20 + v_1 - v_2 = 0$$

Substituting v_1 and v_2, we obtain:

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \quad \Rightarrow \quad i = 4 \text{ A}$$

Substituting i:

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

Example 10 (Homework): Find v_1 and v_2 in the circuit of Fig. 1.21

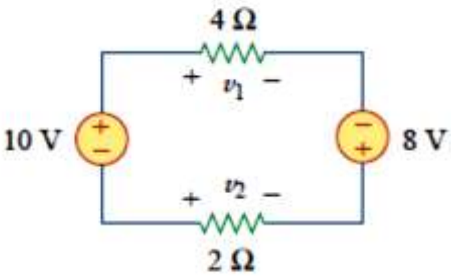


Figure 1.21: Circuit of Example 10. **Answer:** 12 V, - 6 V.

Example 11: Determine *vo* and *i* in the circuit shown in Fig. 1.22(a).

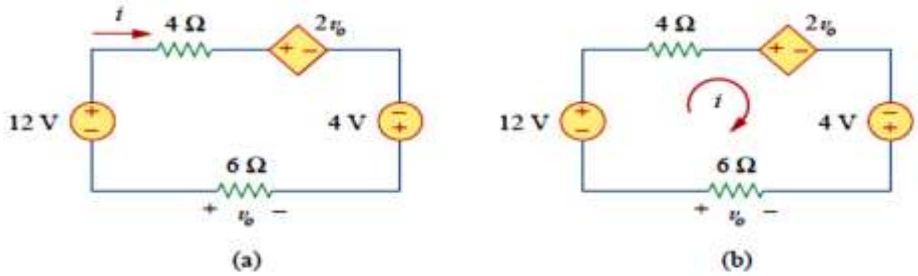


Figure 1.22: Circuit of Example 11.

Solution: We apply KVL around the loop as shown in Fig. 1.22(b). The result is:

$$-12 + 4i + 2v_o - 4 + 6i = 0$$

Applying Ohm's law to the 6-Ω resistor gives:

$$v_o = -6i$$

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Example 12: Find current ***io*** and voltage ***vo*** in the circuit shown in Fig. 1.23.

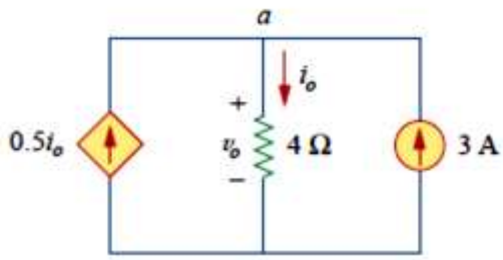


Figure 1.23: Circuit of Example 12.

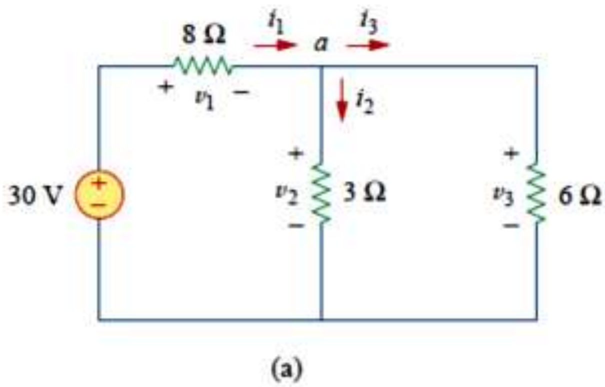
Solution: Applying KCL to node a, we obtain:

$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

For the 4Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

Example 13: Find currents and voltages in the circuit shown in Fig. 1.24(a).



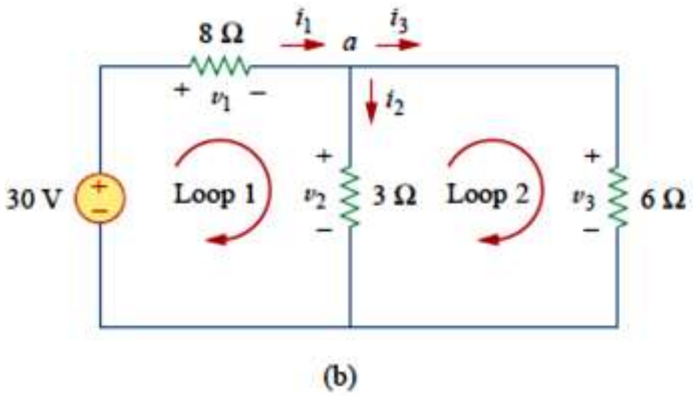


Figure 1.24: Circuit of Example 13.

Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for:

(v_1, v_2, v_3) or (i_1, i_2, i_3)

At node a, KCL gives:

$$i_1 - i_2 - i_3 = 0$$

Applying KVL to loop 1 as in Fig. 1.23(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i1 and i2:

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Rightarrow \qquad v_3 = v_2$$

Since

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3$$

If we express ***v1*** and ***v2*** in terms of ***i1*** and ***i2*** we obtain:

$$6i_3 = 3i_2 \qquad \Rightarrow \qquad i_3 = \frac{i_2}{2}$$

so

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$\therefore \; i_2 = 2$ A

From the value of ***i2***, we now use the above equations to obtain:

$i_1 = 3$ A, $\quad i_3 = 1$ A, $\quad v_1 = 24$ V, $\quad v_2 = 6$ V, $\quad v_3 = 6$ V

Example 14 (Homework): Find the currents and voltages in the circuit shown in Fig. 1.25.

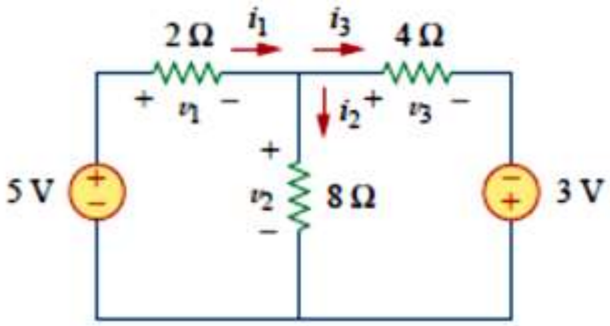


Figure 1.25: Circuit of Example 14

Answer: $v_1 = 3$ V, $v_2 = 2$ V, $v_3 = 5$ V, $i_1 = 1.5$ A, $i_2 = 0.25$ A, $i_3 = 1.25$ A.

2.4. Series Resistors and Voltage Division

Consider the single-loop circuit of Fig. 1.26.

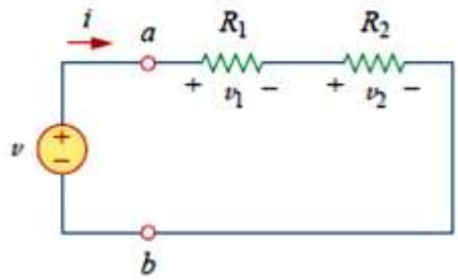


Figure 1.26: A single-loop circuit with two resistors in series.

The two resistors are in series . Applying Ohm's law to each of the resistors, we obtain:

$$v_1 = iR_1, \qquad v_2 = iR_2$$

If we apply KVL to the loop (moving in the clockwise direction), we have:

$$-v + v_1 + v_2 = 0$$

so

$$v = v_1 + v_2 = i(R_1 + R_2)$$

Or

$$i = \frac{v}{R_1 + R_2}$$

$$v = iR_{eq}$$

Where:

$$R_{eq} = R_1 + R_2$$

∴The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

**Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the
individual resistors.**

To determine the voltage across each resistor in Fig. 1.25:

$$v_1 = \frac{R_1}{R_1 + R_2}v, \qquad v_2 = \frac{R_2}{R_1 + R_2}v$$

Notice that the source voltage ***v*** is divided among the resistors in direct proportion to their resistances; the ***larger the resistance***, the ***larger the voltage drop***. This is called the principle of ***voltage division.*** In general:

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N}v$$

2.5. Parallel Resistors and Current Division

Consider the circuit in Fig. 1.27:

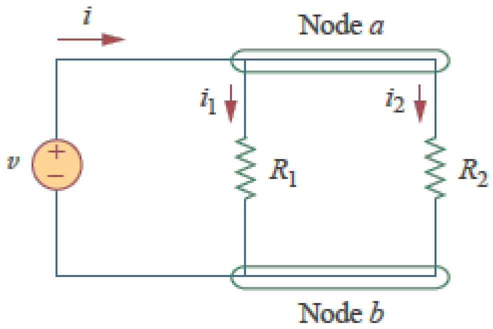


Figure 1.27: Two resistors in parallel.

The two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1R_1 = i_2R_2$$

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

Applying KCL at node ***a*** gives the total current ***i*** as:

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

where $\mathbf{R_{eq}}$ is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1R_2}$$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

In general, a circuit with N resistors in parallel. The equivalent resistance is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination.

Given the total current ***i*** entering node a in Fig. 1.27, how do we obtain current i_1 and i_2. We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$$

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

This is known as the principle of current division, and the circuit in Fig. 1.25 is known as a current divider.

Notice that the larger current flows through the smaller resistance.

Example 15: Find R_{eq} for the circuit shown in Fig. 1.28.

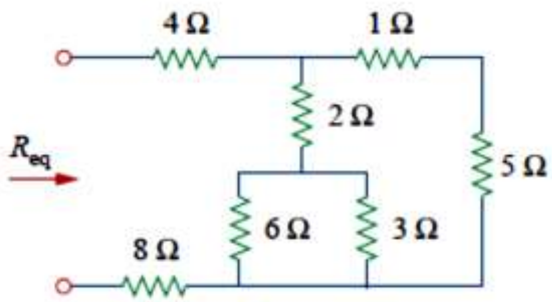


Figure 1.28: Circuit of Example 15

The 6Ωand 3Ω resistors are in parallel, so their equivalent resistance is:

$$6\,\Omega \,\|\, 3\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

The symbol ‖ is used to indicate a parallel combination. Also, the 1Ω and 5Ω resistors are in series

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

Thus the circuit in Fig. 1.27 is reduced to that in Fig. 1.28, in which we notice that the two 2Ω resistors are in series, so the equivalent resistance is:

$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

This 4Ω resistor is now in parallel with the 6Ω resistor, their equivalent resistance is:

$$4\,\Omega \,\|\, 6\,\Omega = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

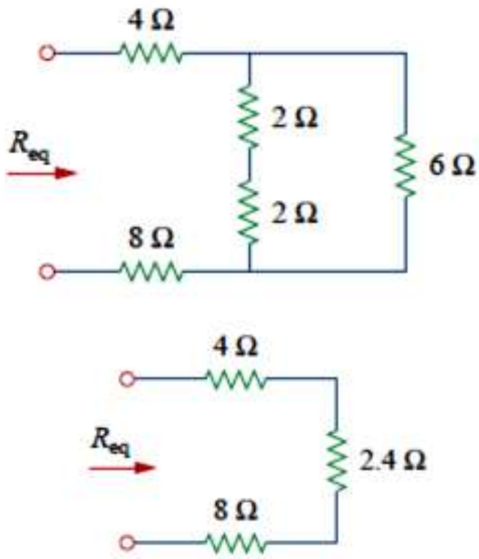


Figure 1.29: Equivalent circuits for Example 14.

Hence, the equivalent resistance for the circuit is:

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Example 16 (Homework): By combining the resistors in Fig. 1.30, find $\mathbf{R_{eq}}$

Answer: 6 Ω

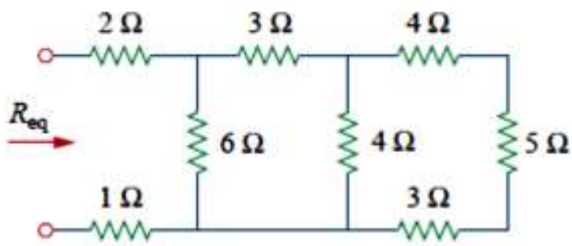


Figure 1.30: Equivalent circuits for Example 16.

Example 17 (Homework): Calculate the equivalent resistance $\mathbf{R_{ab}}$ in the circuit in Fig. 1.29.

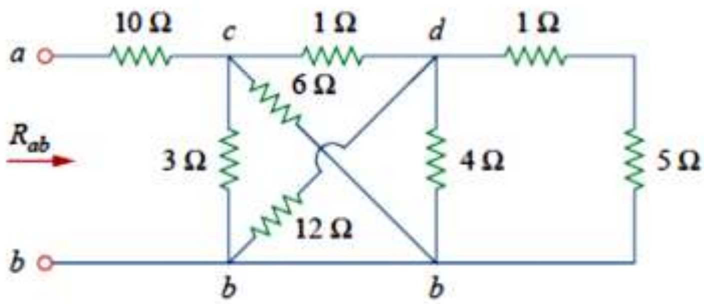


Figure 1.31: Circuits for Example 17.

Answer: R_{ab} = 11.2 Ω

Example 18 (Homework): Calculate the equivalent resistance R_{ab} in the circuit in Fig. 1.32.

Answer: R_{ab} = 11 Ω

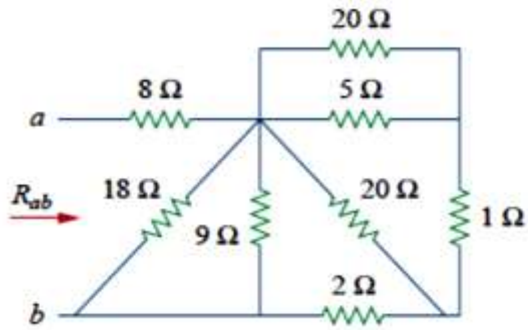


Figure 1.32: Circuit for Example 18.

Example 19 (Homework): Find *io* and *vo* in the circuit shown in Fig.1.33. Calculate the power dissipated in the 3Ω resistor.

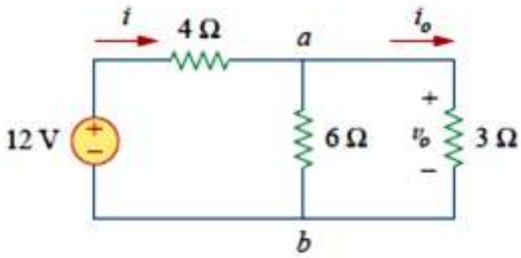


Figure 1.33: Circuit for Example 19.

Answer: *vo* = 4V, *io* = (4/3) A, *Po* = 5.333W

Example 20 (Homework):Find v1 and v2 in the circuit shown in Fig. 1.34. Also calculate i1 and i2 and the power dissipated in the 12Ω and 40Ω resistors.

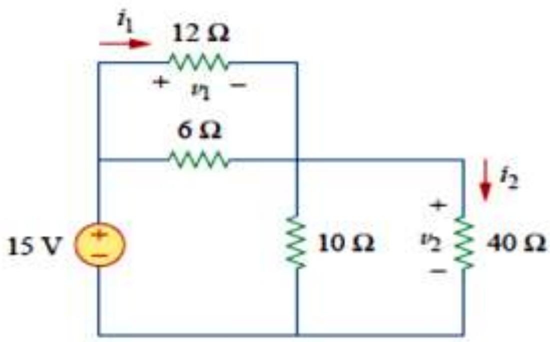


Figure 1.34: Circuit for Example 20.

Example 21 (Homework):For the circuit shown in Fig. 1.35, determine: (a) the voltage *vo* (b) the power supplied by the current source, (c) the power absorbed by each resistor.

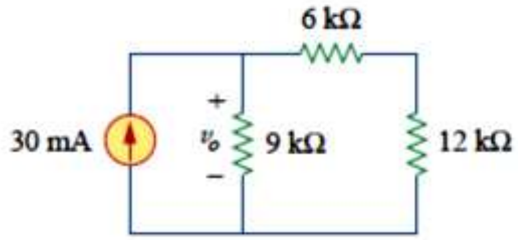


Figure 1.35: Circuit for Example 21.

Answer:

a) vo = 180V
b) b) po = 5.4W,
c) Power absorbed by the 12kΩ resistor = 1.2W
Power absorbed by the 6kΩ resistor = 0.6W
Power absorbed by the 9kΩ resistor = 3.6W

Example 22 (Homework): For the circuit shown in Fig. 1.36, find: (a) v_1 and v_2 (b) the power dissipated in the 3kΩ and 20kΩ resistors, and (c) the power supplied by the current source.

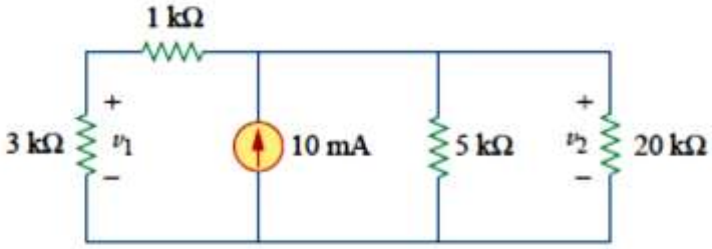


Figure 1.36: Circuit for Example 22.

Answer: (a) 15 V, 20 V, (b) 75 mW, 20 mW, (c) 200 mW.