1. Basic Concepts and Laws

1.1. Introduction

An electric circuit is an interconnection of electrical elements. Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this course is the analysis of the circuits. By the **analysis of a circuit**, we mean **a study of the behavior of the circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact?**

We start our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the course.

1.2. Systems of Units

As engineers, we deal with measurable quantities. Our measurement, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country where the measurement is conducted. Such an international measurement language is the **International System of Units** (SI), adopted by the General Conference on Weights and Measures in 1960.

Table 1.1 shows the six units, their symbols, and the physical quantities they represent. Table 1.2 shows the SI prefixes and their symbols

TABLE 1.1

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

1.3. Charge and Current

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge ***e*** on an electron is negative and equal in magnitude to 1.602×10^{-19}C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

TABLE 1.2

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Electric current is the time rate of change of charge, measured in amperes (A). Mathematically, the relationship between current i, charge q, and time t is:

$$\boldsymbol{i = \frac{dq}{dt} \Rightarrow q = \int_{t1}^{t2} i\, dt}$$

where current is measured in amperes (A), and
1 ampere = 1 coulomb/second

- **A direct current** (dc) is a current that remains constant with time. the symbol (**I**) is used to represent such a constant current

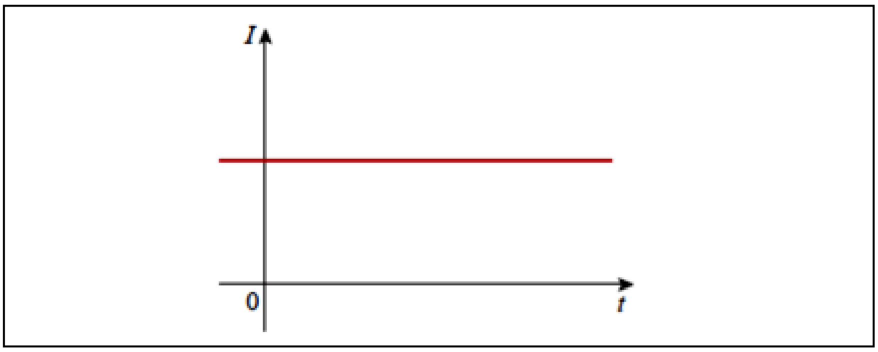


Figure 1.1 Direct current (dc)

- An alternating current (ac) is a current that varies sinusoidally with time. A time-varying current is represented by the symbol (i).

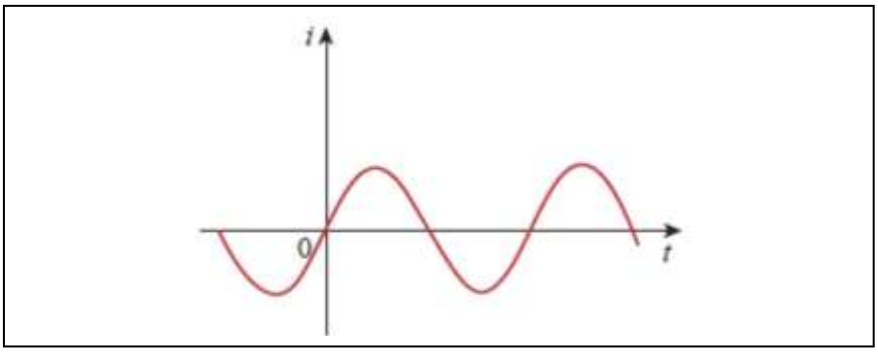


Figure 1.2 Alternating current (ac).

Example 1: Determine the total charge entering a terminal between t = 1 sec. and t = 2 sec. if the current passing the terminal is:

$$i = (3t^2 - t)A$$

Solution:

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5\ \text{C}$$

Example 2: The total charge entering a terminal is given by $\mathbf{q = 5t \sin 4\pi\, t\ mC}$. Calculate the current at t = 0.5s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t)\ \text{mC/s}$$

$$= (5 \sin 4\pi t + 20\pi t \cos 4\pi t)\ \text{mA}$$

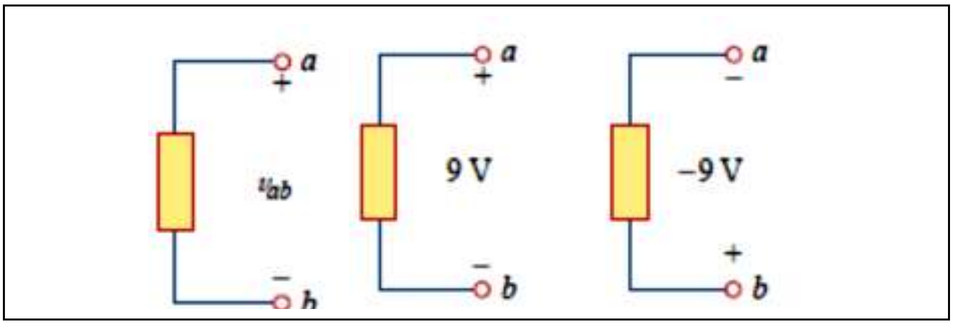
At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42\ \text{mA}$$

1.4. Voltage

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V). mathematically:

$$v = \frac{dw}{dq}$$



a) b) c)

Figure 1.3 a) Polarity of voltage *vab*, Two equivalent representations of the same voltage : (b) point *a* is 9 V above point *b*, (c) point *b* is -9 V above point *a*.

The *vab* can be interpreted in two ways: (1) point *a* is at a potential of *vab* volts higher than point *b*, or (2) the potential at point *a* with respect to point *b* is . It follows logically that in general

$$v_{ab} = v_a - v_b \text{ and } v_{ab} = -v_{ba}$$

1.5. Power and Energy

Power is the time rate of absorbing energy, measured in watts (W).

$$p = \frac{dw}{dt}$$

where *p* is power in watts (W), *w* is energy in joules (J), and *t* is time in seconds (s).

$$p = \frac{dw}{dt} \times \frac{dq}{dq} = \frac{dw}{dq} \times \frac{dq}{dt} = vi$$

the power absorbed or supplied by an element is the product of the voltage across the element and the current through it.

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where:

1 Wh = 3,600 J

Example 3: How much energy does a 100-W electric bulb consume in two hours?

Solution:

$w = pt = 100\text{ W} \times 2\text{ h} = 200\text{ Wh}$

Or

$w = pt = 100\text{ W} \times 2\text{ h} \times 60\text{ (min/h)} \times 60\text{ (s/min)}$

$= 720000\text{ J} = 72\text{kJ}$

1.6. Circuit Elements

An electric circuit is simply **an interconnection of the electric elements**. Circuit analysis is the **process of determining voltages across (or the currents through) the elements of the circuit**.

There are two types of elements found in electric circuits:

- **Passive elements (cannot generate energy)**

Examples of passive elements are resistors, capacitors, and inductors

- **Active elements (capable of generating energy)**

Typical active elements include generators, batteries, and operational amplifiers. The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

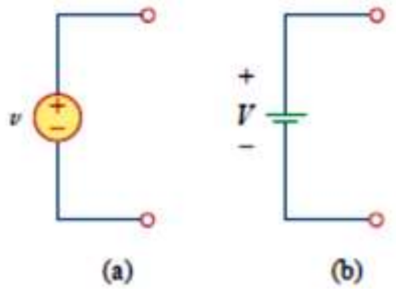


Figure 1.4 Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

Figure 1.5 Symbol for independent current source

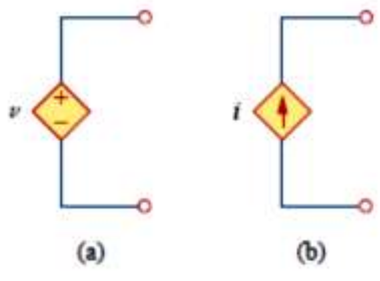


Figure 1.6 Symbols for: (a) dependent voltage source, (b) dependent current source.

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

2. Basic Laws

2.1. Ohm's Law

Ohm's law states that: "**The voltage V across a resistor is directly proportional to the current I flowing through the resistor**".

$$\boldsymbol{V \propto I\ ,\ V = IR} \qquad \textit{Ohms Law}$$

Where R is the resistance.

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

Ohms Law can also be written as:

$$\boldsymbol{R = \frac{V}{I}}\text{, so that: } 1\ \boldsymbol{\Omega} = 1\ \text{V/A}$$

For any material, The resistance R depends on its physical dimensions as follows:

$$R = \rho \frac{L}{A}$$

where ρ is known as the resistivity of the material in ohm-meters.

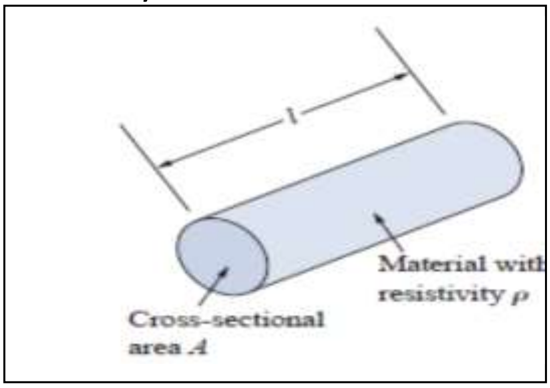
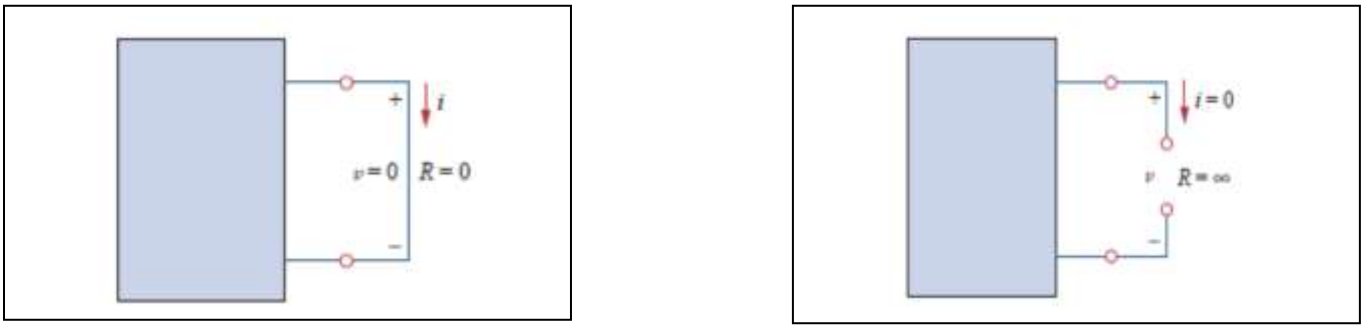


Figure 1.7 Resistor

Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities.

- **A short circuit is a circuit element with resistance approaching zero.**
- **An open circuit is a circuit element with resistance approaching infinity**



(a) **(b)**

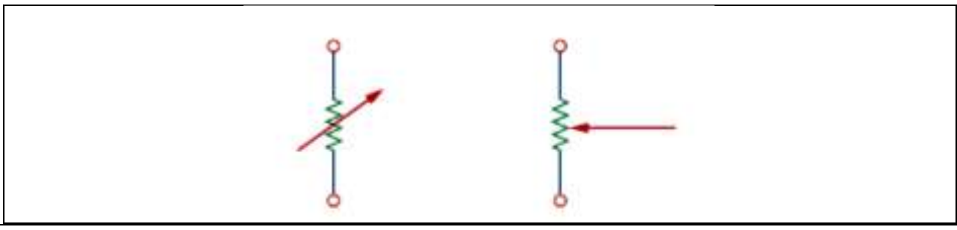
Figure 1.8 (a) Short circuit (R=0), (b) Open circuit (R= ∞)

A resistor is either fixed or variable. Most resistors are of the fixed type, meaning their resistance remains constant.

TABLE 2.1

Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator



(a) **(b)**

Figure 1.9 Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

A useful quantity in circuit analysis is the reciprocal of resistance R, known as conductance and denoted by G:

$$G = \frac{1}{R} = \frac{i}{v}$$

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the mho (ohm spelled backward) or reciprocal ohm, with symbol (℧), the inverted omega or Siemens (S).

$$1\ \text{S} = 1\ ℧ = 1\ \text{A/V}$$

Thus,

Conductance (G) is the ability of an element to conduct electric current; it is measured in mhos (℧) or Siemens (S).

$$i = Gv$$

The power dissipated by a resistor can be expressed in terms of R.

$$p = vi = i^2R = \frac{v^2}{R}$$

$$p = vi = v^2G = \frac{i^2}{G}$$

Example 4: In the circuit shown in Fig. 1.10, calculate the current i, the conductance G, and the power p.

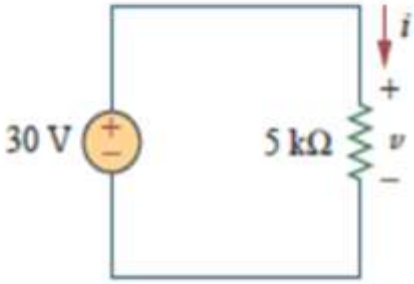


Figure 1.10. Circuit of Example 4

Solution:

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways:

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

Or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

Or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Example 5 (Homework): For the circuit shown in Fig. 1.11, calculate the voltage v, the conductance G, and the power p.

Figure 1.11. Circuit of Example 5

Answer: **20 V, 100 mS, 40 mW.**

Example 6 (Homework): The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 10 Ω at 110 V? **Answer:** 11 A.

2.2. Nodes, Branches, and Loops

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology.

- **A branch represents a single element such as a voltage source or a resistor.**
- **A node is the point of connection between two or more branches.**
- **A loop is any closed path in a circuit**

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be ***independent*** if it contains at least one branch which is not a part of any other independent loop

- **Two or more elements are in series if they exclusively share a single node and consequently carry the same current.**
- **Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.**

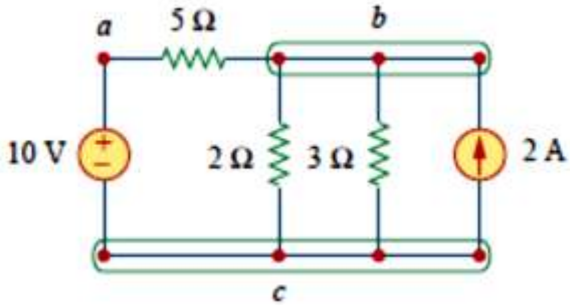


Figure 1.12. Nodes, branches, and loops.

The circuit in Fig. 1.12 has three nodes a, b and c. Notice that the three points that form node b are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node c.

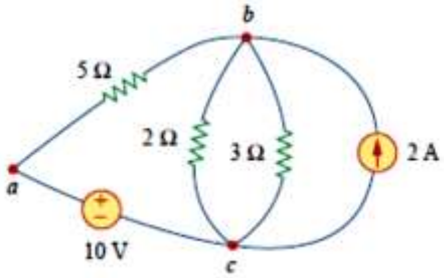


Figure 1.13. The three-node circuit of Fig. 1.12 is redrawn.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

Example 7: Determine the number of branches and nodes in the circuit shown in Fig. 1.14. Identify which elements are in series and which are in parallel.

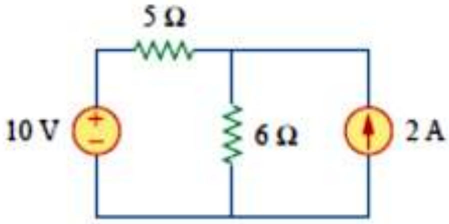


Figure 1.14. Circuit of Example 7

Solution:

- ✓ **the circuit has four branches:**

10 V, 5Ω , 6Ω and 2 A.

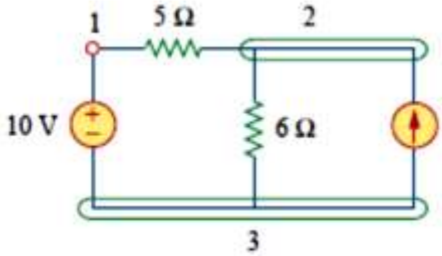


Figure 1.15. The three nodes in the circuit of Fig. 1.14.

- ✓ **The circuit has three nodes as identified in Fig. 1.14.**
- ✓ **The 5Ω resistor is in series with the 10V voltage source because the same current would flow in both.**
- ✓ **The 6Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.**

Example 8: How many branches and nodes does the circuit in Fig. 1.16 have? Identify the elements that are in series and in parallel.

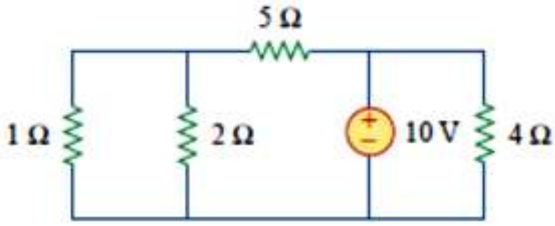


Figure 1.16. Circuit of example 8.

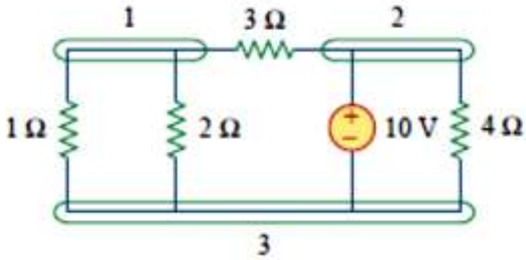


Figure 1.17. Answer for example 8.

Solution:

- ✓ **5 branches, 3 nodes are identified in Fig. 1.17**
- ✓ **The 1Ω and 2Ω resistors are in parallel.**
- ✓ **The 4Ω resistor and 10V source are also in parallel.**