3.2. Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. **A mesh is a loop that does not contain any other loop within it**. Mesh analysis applies KVL to find unknown currents.

Steps to Determine Mesh Currents:

1. Assign mesh currents i_1, i_2, …., i_n to the *n* meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Example 1:

For the circuit in Fig. 1, find the branch currents I_1, I_2 and I_3 using mesh analysis.

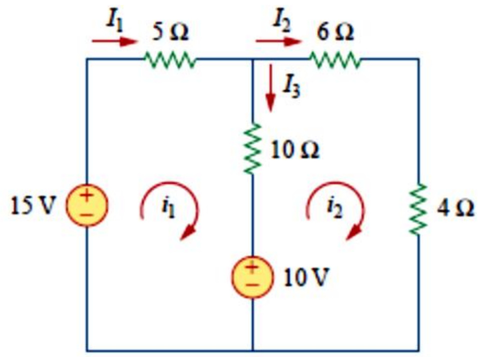

Figure 1.

Solution: We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1$$

For mesh 2

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1$$

Substituting the above two equations we get:

$$6i_2 - 3 - 2i_2 = 1 \quad \Rightarrow \quad i_2 = 1\text{ A}$$

From last equation:

$$i_1 = 2i_2 - 1 = 2 - 1 = 1\text{ A. Thus,}$$

$$I_1 = i_1 = 1\text{ A}, \qquad I_2 = i_2 = 1\text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

Example 2 (Homework): Calculate the mesh currents i_1 and i_2 of the circuit of Fig. 2.

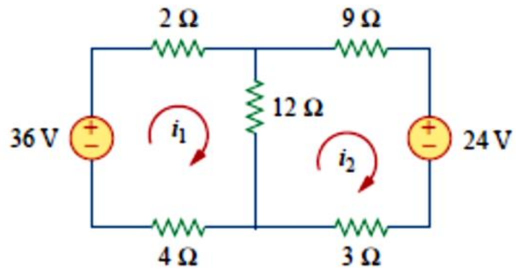

Figure 2.

Answer: $i_1 = 2$ A, $i_2 = 0$ A.

Example 3: Use mesh analysis to find the current in the circuit of Fig. 3.

Solution: We apply KVL to the three meshes in turn.

- **For mesh 1,**

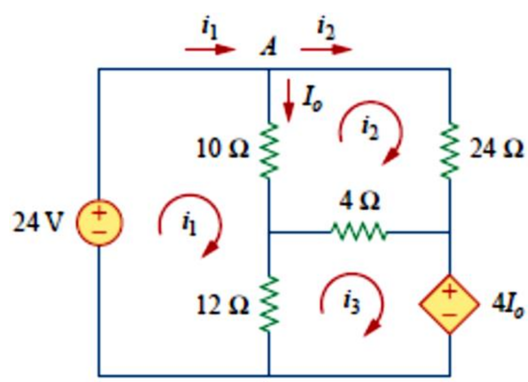

Figure 3.

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12$$

- **For mesh 2,**

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0$$

- **For mesh 3,**

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0$$

In matrix form, the above three Equations, become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as:

$$\Delta = \begin{array}{c|ccc|c} & 11 & -5 & -6 & \\ & -5 & 19 & -2 & \\ & -1 & -1 & 2 & \\ - & 11 & -5 & -6 & + \\ - & -5 & 19 & -2 & + \\ - & & & & + \end{array}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{array}{c|ccc|c} & 12 & -5 & -6 & \\ & 0 & 19 & -2 & \\ & 0 & -1 & 2 & \\ - & 12 & -5 & -6 & + \\ - & 0 & 19 & -2 & + \\ - & & & & + \end{array} = 456 - 24 = 432$$

$$\Delta_2 = \begin{array}{c|ccc|c} & 11 & 12 & -6 & \\ & -5 & 0 & -2 & \\ & -1 & 0 & 2 & \\ - & 11 & 12 & -6 & + \\ - & -5 & 0 & -2 & + \\ - & & & & + \end{array} = 24 + 120 = 144$$

$$\Delta_3 = \begin{array}{c|ccc|c} & 11 & -5 & 12 & \\ & -5 & 19 & 0 & \\ & -1 & -1 & 0 & \\ - & 11 & -5 & 12 & + \\ - & -5 & 19 & 0 & + \\ - & & & & + \end{array} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as:

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

Example 4 (Homework):

Using mesh analysis, find I_0 in the circuit of Fig. 4.

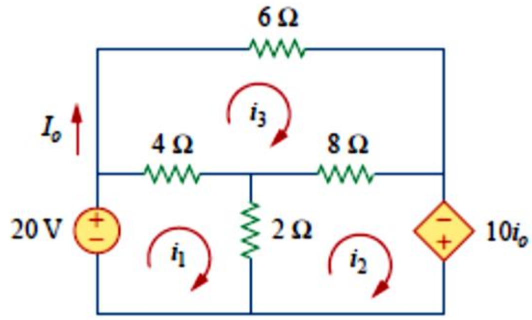

Figure 4.

Answer: −5 A.

3.3. Superposition

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the **superposition**. The idea of superposition rests on the linearity property. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.

Example 5:

Use the superposition theorem to find *v* in the circuit of Fig. 5.

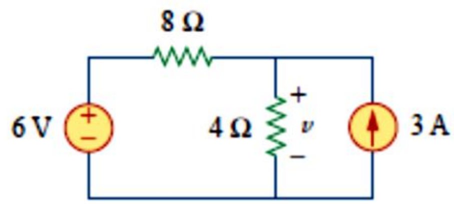

Figure 5.

Solution:

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6V voltage source and the 3A current source, respectively. To obtain v_1 we set the current source to zero, as shown in Fig. 6(a). Applying KVL to the loop in Fig. 6(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

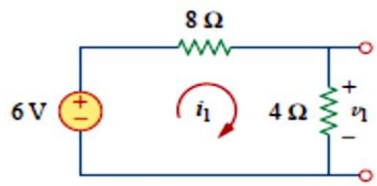

Figure 6(a).

Thus:

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2 we set the voltage source to zero, as in Fig. 6(b). Using current division

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

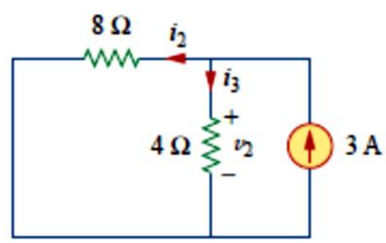

Figure 6(b).

Hence:

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10\text{ V}$$

Example 6 (Homework): Using the superposition theorem, find v_o in the circuit of Fig. 7.

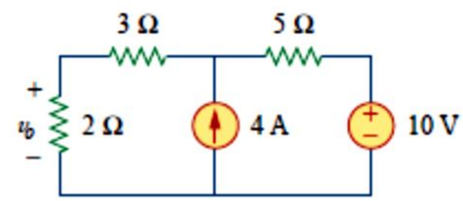

Figure 7.

Answer: 6 V.

Example 7: For the circuit in Fig. 8, use the superposition theorem to find i.

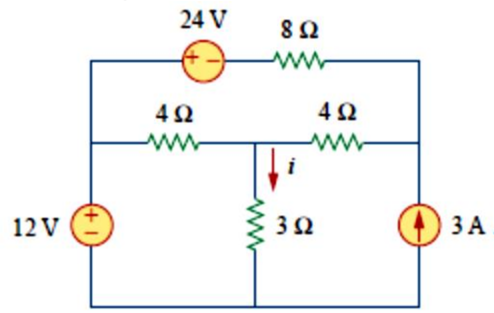

Figure 8.

Solution:

In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

Where: i_1 is due to 12V source, i_2 is due to 24V source and i_3 is due 3A source. To get i_1, consider the circuit in Fig. 9.(a).

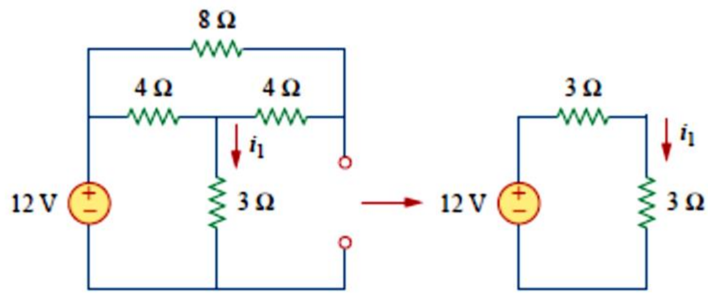

Figure 9 (a)

8Ω + 4Ω (on the right hand side) = 12Ω (in series)
12Ω // 4Ω = (12Ω × 4Ω / (12Ω + 4Ω)) = 3Ω
Thus,

$$i_1 = \frac{12}{6} = 2\text{ A}$$

To get i_2, consider the circuit in Fig. 9.(b). Applying mesh analysis gives:

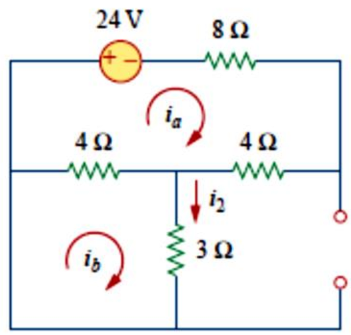

Figure 9 (b)

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b$$

Substituting last equation with its previous gives:

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 9(c). Using nodal analysis gives

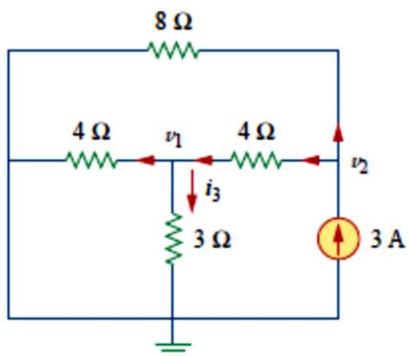

Figure 9 (c)

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1$$

Substituting last equation with its previous leads to v_1=3, and

$$i_3 = \frac{v_1}{3} = 1\text{ A}$$

Thus

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2\text{ A}$$

Example 8 (Homework): Find I in the circuit of Fig. 10 using the superposition principle.

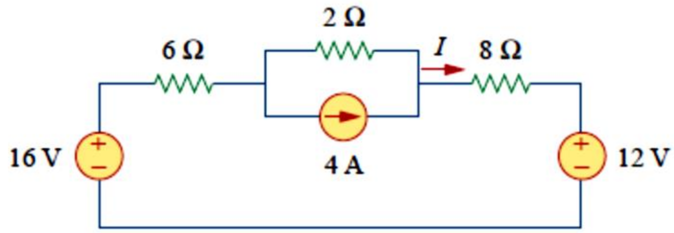

Figure 10

Answer: 0.75 A.

3.4. Source Transformation

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. Source transformation is another tool for simplifying circuits. Basic to these tools is the concept of equivalence. We recall that an equivalent circuit is one whose v-i characteristics are identical with the original circuit.

A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

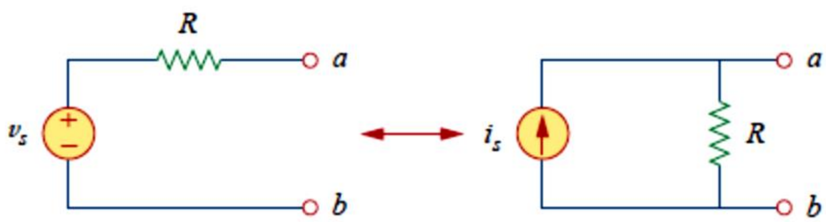

Figure 11: Transformation of independent sources

The two circuits in Fig. 11 are equivalent—provided they have the same voltage-current relation at terminals a-b. It is easy to show that they are indeed equivalent. If the sources are turned off, the equivalent resistance at terminals a-b in both circuits is R. Also, when terminals are short-circuited, the short-circuit current flowing from a to b is $i_{sc} = v_s / R$, in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the right-hand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.

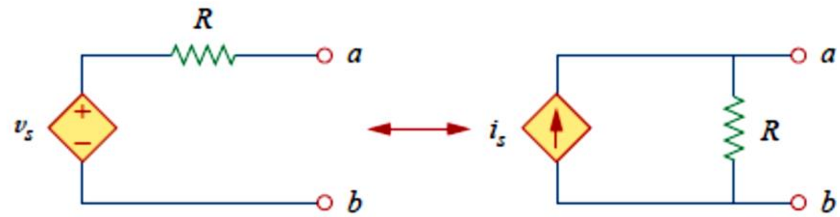

Figure 12: Transformation of dependent sources

However, we should keep the following points in mind when dealing with source transformation.

1. Note from Fig. 11 (or Fig. 12) that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from transformation equation that source transformation is not possible when R=0, which is the case with an ideal voltage source. However, for a practical, non-ideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source.

Example 9: Use source transformation to find v_o in the circuit of Fig. 13.

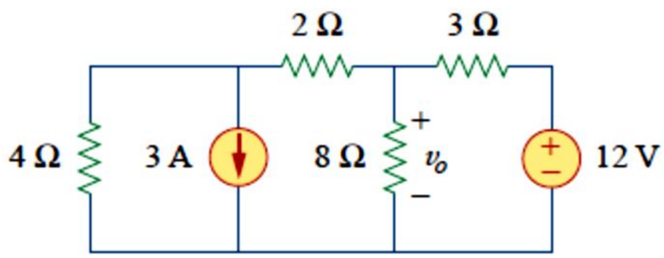

Figure 13

Solution:

We first transform the current and voltage sources to obtain the circuit in Fig. 14(a).

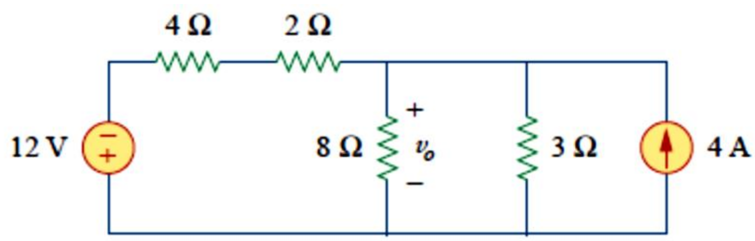

Figure 14(a)

$4\Omega + 2\Omega = 6\Omega$ (in series)

transforming the 12-V voltage source gives us Fig. 14(b).

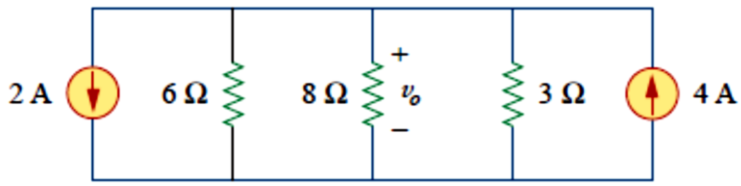

Figure 14(b)

$3\Omega /\!/ 6\Omega = (3\Omega \times 6\Omega) / (3\Omega + 6\Omega) = 2\Omega$

We also combine the 2A and 4A current sources to get a 2A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 14(c).

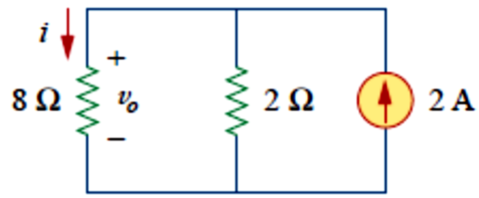

Figure 14(c)

We use current division in Fig. 14(c) to get:

$$i = \frac{2}{2+8}(2) = 0.4 \text{ A}$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8Ω and 2 Ω resistors in Fig. 14(c) are in parallel, they have the same voltage across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

<u>Example 10 (Homework):</u>

Find i_o in the circuit of Fig. 15 using source transformation.

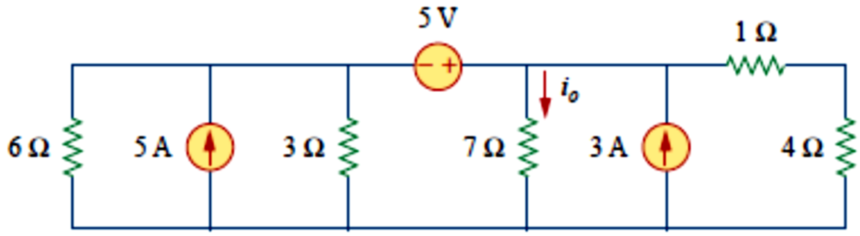

Figure 15

Answer: 1.78 A.

Current Sources In Parallel

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction.

Example 11: Reduce the parallel current sources of Figs. 16 and 17 to a single current source.

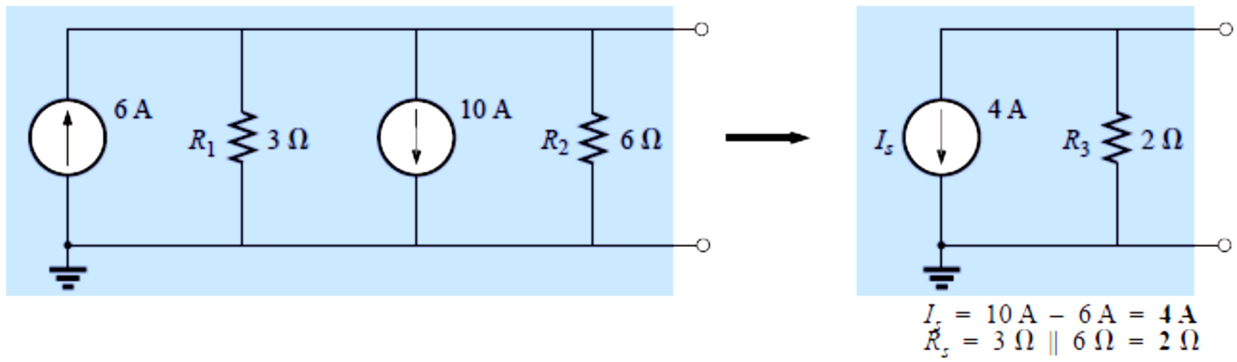

Figure 16

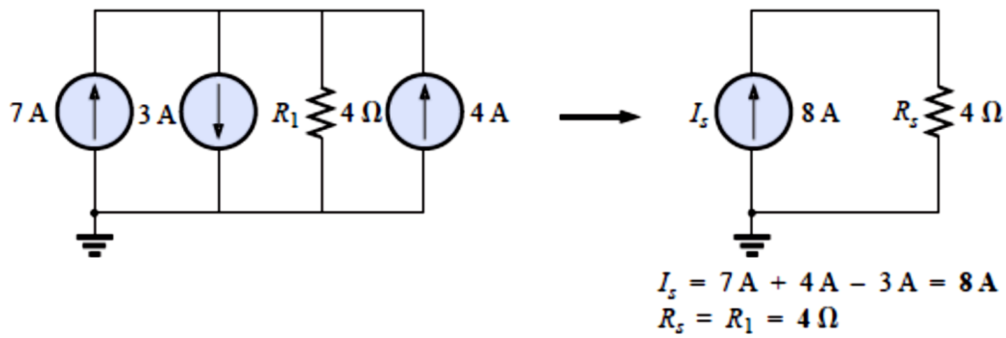

Figure 17

Example 11: Reduce the network of Fig. 18 to a single current source, and calculate the current through RL.

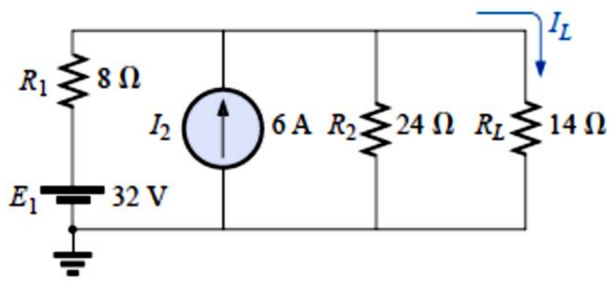

Figure 18

Solution:

The voltage source will first be converted to a current source as shown in Fig. 19.

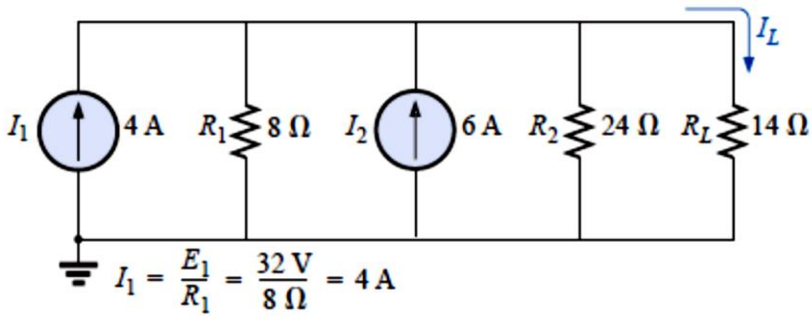

Figure 19

Combining current sources, and finding the equivalent resistance results in Fig. 20.

$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\ A}$$

$$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

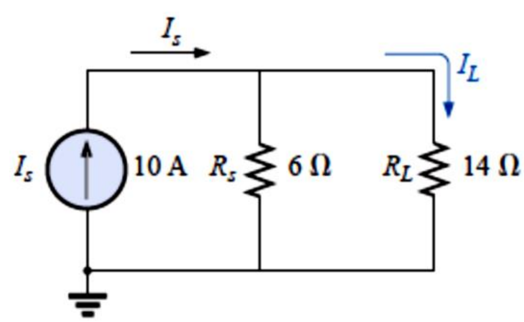

Figure 20

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6\ \Omega)(10\text{ A})}{6\ \Omega + 14\ \Omega} = \frac{60\text{ A}}{20} = \mathbf{3\ A}$$

<u>Example 12:</u> Determine the current I_2 in the network of Fig. 21.

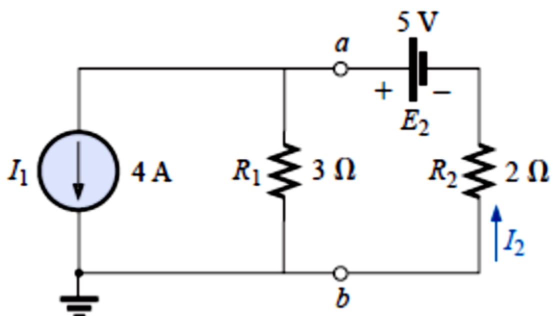

Figure 21

<u>Solution:</u>

$$E_s = I_1 R_1 = (4\text{ A})(3\ \Omega) = 12\text{ V}$$

$$R_s = R_1 = 3\ \Omega$$

$$I_2 = \frac{E_s + E_2}{R_s + R_2} = \frac{12\text{ V} + 5\text{ V}}{3\ \Omega + 2\ \Omega} = \frac{17\text{ V}}{5\ \Omega} = \mathbf{3.4\ A}$$

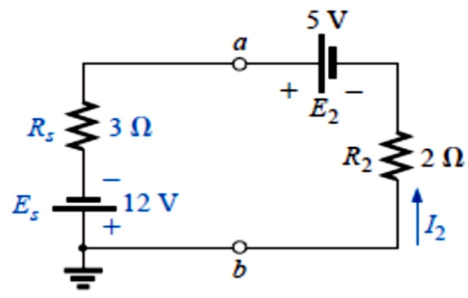

Figure 22